Week 3 Section IV Chapter 9 Fourier Series

You will often notice that some top hits are integer multiples of a frequency at more powerful periodogram peak (or very close approximation thereto). These are harmonics of the lower frequency. We have touched on harmonics earlier in this week's material dealing with periodograms and also in week 1 and the week 2 material on periodicity. Earlier this week in Exercise 2 of section II on periodograms we ran head-on into a forest of harmonics performing the DCDFT of the Bet Per light curve. In week 1 we created a model of Del Cep mimicking the smoothed curve in section 1.4 of Analyzing Light Curves using a limited number of harmonics. A fundamental frequency together with a set of higher order harmonics used to create a periodic non-sinusoidal wave shape is called a Fourier series. In Our bet Per exercise we saw may harmonics in the light curve resulting from the simple orbital motion of two stars (Our analysis didn't explore the non-periodic jumps in period or the much longer period effects of the third star, Algol C). Any periodic relationship can be represented by sums of harmonics but it may take a very large or even infinite number of harmonics to exactly duplicate the shape of the light curve. However, we can come as close as we want to duplicating the periodic function by including a sufficient number of harmonics as in the saw-tooth waveform example in Chapter 9 of Analyzing Light Curves. Therefore, just because a periodic signal isn't sinusoidal or even "smooth" doesn't mean it can't be represented by the sums of sinusoids. However, It may require several, perhaps many sinusoids added together to come as close to the signal as we would like. The Fourier series representation of the sawtooth waveform still had visible "spikes" (actually, ringing) at the corners where the slopes break sharply (where the derivatives of the function do not exist) when approximated by a Fourier series containing 100 harmonics. The approximation using 1000 harmonics was indistinguishable by eye from the perfect sawtooth waveform, but if you magnified the region near the corners sufficiently, you would see that it still does not reproduce the corners perfectly. If you used 10,000 harmonics you would need even greater magnification to see the deviation from the actual sawtooth. In the physical world, discontinuities in the waveform or the slope of the waveform rarely happen. That requires infinitely great forces. Therefore a physical process with a sawtooth-like waveform would actually only be approximately a sawtooth waveform, the del Cep light curve, for example.

In VStar you can include harmonics in a model by picking them out of the top hits. However, it is better to include them (up to a maximum of 12) using the harmonics drop down list of the model dialog on the DCDFT Top Hits tab. This has several advantages. Obviously you don't have to worry about accidentally leaving out a harmonic. Harmonics included in top hits are not at exactly the correct frequency due to resolution limits of the scan but you want them to be the precise harmonics. Finally, you have to do less scrolling and clicking. It is also best to use the output of CLEANest run on the fundamental frequency(s) to refine the frequency determination of the fundamental and therefore the harmonics. A small difference in the fundamental frequency is amplified as the harmonic number increases. Also remember that the default value of "1" in the drop down box is the fundamental itself. The number you select is the number of the highest harmonic that will be included in the least squares model fit.

The Top Hits tab in DCDFT has a button for finding harmonics but it does not work reliably. The reason for this has not yet been determined. Therefore, don't use that button for now. Search for harmonics in the drop down list. You can also make several models with different maximum.

When referring to harmonics please do not use the term "overtones." These terms may be synonymous when discussing music but they usually mean different things when discussing light curves. The term "overtones" generally refers to different modes of pulsation rather than harmonics.

In pulsating stars, individual modes of pulsation may or may not have prominent harmonics and different modes of pulsation may or may not be harmonically related. Relationships between the pulsation frequencies of different modes in pulsating stars is a subject of asteroseismology and is far beyond the scope of this course. For this course it is sufficient to understand that harmonics present in light curves of pulsating stars usually do not indicate multiple modes of pulsation and if different modes are harmonically related it is just a chance occurrence arising from the specific conditions existing in that particular star. Harmonics in pulsing systems most commonly arise from non-linearity in the physical process (like stretching a spring too far so that the restoring force is not proportional to the distance the spring is stretched). When extreme forces are in play, non-linearity is common, and therefore, harmonics are common.

If for some reason there is doubt whether a signal is a harmonic or arises as an alias of some other frequency you can perform a CLEANest on the other frequency and create a model with the result. If the frequency or frequencies you thought might be an alias of the modeled frequency remains in the residuals it is not an alias. There will be more on interactions between frequencies in Chapter 12.

When reading papers about pulsating stars amplitudes and phases are often reported as relative amplitudes and relative phases. The relative amplitude is just the amplitude of a higher order harmonic divided by the amplitude of the first harmonic (the fundamental) Relative phase of the nth harmonic to the phase of the fundamental is just

$\emptyset_{n1} = \emptyset_n - n\emptyset_1$where $\emptyset_n$ is the phase of the nth harmonic at time $t = 0$ and $\emptyset_1$ is the phase of the fundamental at time $t = 0$. Since it is common to pick a reference epoch $t = \tau$ rather than $t = 0$ so that the phase of the fundamental is 0 at time τ, then $\phi_{n1}$is the phase of the nth harmonic at time $t = \tau$.