Quantifying Irregularity in Pulsating Red Giants

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Abstract Hundreds of red giant variable stars are classified as "type L," which the *General Catalogue of Variable Stars* (GCVS) defines as "slow irregular variables of late spectral type...which show no evidence of periodicity, or any periodicity present is very poorly defined...." Self-correlation (Percy and Muhammed 2004) is a simple form of time-series analysis which determines the cycle-to-cycle behavior of a star, averaged over all the available data. It is well suited for analyzing stars which are not strictly periodic. Even for non-periodic stars, it provides a "profile" of the variability, including the average "characteristic time" of variability. We have applied this method to twenty-three L-type variables which have been measured extensively by AAVSO visual observers. We find a continuous spectrum of behavior, from irregular to semiregular.

1. Introduction

Cool red giants are all variable in brightness; there are a dozen excellent essays on Mira and Red Semiregular variables in the "Variable Star of the Season" archive on the AAVSO website. The basic cause of the variability is pulsation. On average, the period and the amplitude increase with decreasing temperature (and increasing size, since the stars are evolving up the giant branch or asymptotic giant branch in the H-R diagram). The coolest (late M spectral type) red giants have visual amplitudes greater than 2.5 and are classified as Mira variables. Less-cool red giants, with visual amplitudes less than 2.5, are placed in one of two classes in the *General Catalogue of Variable Stars*, or GCVS (Kholopov *et al.* 1985): semiregular (SR), or irregular (L).

These types are defined thus:

- SR: "... giants or supergiants of intermediate and late spectral types showing noticeable periodicity in their light changes, accompanied or

sometimes interrupted by various irregularities....”

- L: “Slow irregular variables. The light variations of these stars show no evidence of periodicity, or any periodicity present is very poorly defined, and appears only occasionally....”

Clearly these definitions are qualitative at best; is there a boundary between these two classes? Classification is generally made from light curves, often with limited data.

Kiss *et al.* (1999) have carried out a comprehensive time-series analysis of AAVSO visual observations of a large sample of SR variables; most have one period, some have two periods, and a few have three periods. Their sample included five L-type variables: AA Cas, DM Cep, TZ Cyg, V930 Cyg, and CT Del. They found a period of 367 days for DM Cep (possibly an artifact), 247 days for V930 Cyg, and 138 and 79 days for TZ Cyg, and apparently no periods for the other two stars.

We have found that, for stars with appreciable irregularity, self-correlation (Percy and Mohammed 2004 and references therein) is a useful method of time-series analysis. The purpose of the present paper is to apply this method to the study of L-type pulsating red giants.

2. Sources of data

Visual measurements of the twenty-three L stars listed in Table 1 came from the AAVSO International Database, spanning up to a century. There are dozens of L-type red giant variables in the database but, for most of them, the data are sparse. We have chosen stars which had at least 100 observations available. The precision of visual measurements is known to be about 0.2 to 0.3 magnitude. The intercept on the vertical axis of the self-correlation diagram is a measure of the average precision of the measurements, and is consistent with the estimate above (Figures 1–5: intercepts 0.21 to 0.28 magnitude).

3. Analysis of the stars by self-correlation

Self-correlation is a simple method of time-series analysis that determines the characteristic time scale and amplitude of the variability, averaged over the dataset. For a discussion of its nature, strengths, and weaknesses, see Percy and Mohammed (2004) and references therein. Our self-correlation software is available at: http://www.astro.utoronto.ca/~percy/index.html, and a manual for its use is available at: http://www.astro.utoronto.ca/~percy/manual.pdf.

If a star has any periodic behavior, it will show up as a series of minima in the self-correlation diagram, at multiples of the period. The number of minima is a measure of the coherence of the period. If there are no minima, then the star is considered irregular. If there are a large number of minima, the period

is regular or coherent. So the number of minima is a quantitative measure of the irregularity or semiregularity of the star.

In constructing self-correlation diagrams, one needs to choose Δt (max), the maximum value of Δt (the minimum is always zero), and the number of bins. The stars in our sample are believed to vary on time scales of tens to thousands of days, so that range was used in selecting values for Δt (max). The number of bins should be chosen so that there are ten or more points in each bin, because of the statistical nature of the method.

4. Results

The results are summarized in Table 1. The columns give: the star name, the spectral type (generally from SIMBAD), the approximate total range (from the light curve), the period(s) if any, the number of minima (which is a measure of the coherence of the period), the amplitude, the number of data points, and the total timespan of the data. The stars show a wide spectrum of behavior.

V Aps, PY Cas, TT Leo, GN Her, TY Oph, and WW Cas show no repeating minima in their self-correlation diagrams; they simply rise to a plateau. We consider these stars to be irregular. TY Oph shows one minimum at 200 days, so it may be marginally periodic. Its amplitude is less than 0.05 magnitude.

Most of the other stars show at least two minima, at integral multiples of the period. The number of minima is a measure of the coherence of the period. Stars such as U Ant and X Lyr, with only two minima, can barely be considered as regular. Stars such as AT Dra, with a dozen or more minima, are much more coherent.

U Ant, AT Dra, X Lyr, EX Ori, and τ^4 Ser show a second period, an order of magnitude longer than the first, which would be considered to be a "long secondary period" (LSP). About a third of pulsating red giants show such an LSP; the nature and cause of these LSPs is not known (Wood *et al.* 2004).

For UX Cam and CP Tau especially, it is not clear whether the single period is an LSP (without a primary period) or whether the single period is simply a long primary period.

Self-correlation diagrams are shown for three representative stars: τ^4 Ser (Figure 1) shows a sequence of minima at multiples of 100 days, and also of 1,200 days, the "long secondary period"; OP Her (Figure 2) shows a series of minima, indicating a reasonably coherent period of 75 days; TY Oph (Figure 3) shows no repeating minima, though there is slight evidence for a time scale of about 200 days—maxima at 100 and 300 days, and a minimum at 200 days.

5. Discussion and conclusions

In the course of this research, we have encountered some of the inherent limitations of the self-correlation method.

We occasionally encounter stars, such as UW Dra (Figure 4), which show a very low-amplitude signal at a period of 365.25 days. Apparently, this is a spurious effect that arises because of the way that visual observations are made. JRP first heard of this effect from the late Dr. Janet A. Mattei. It is called the Ceraski effect, and is briefly described by Buchheim (2007) and by Gunther and Schweitzer (undated). The magnitude difference between two stars is perceived differently, depending on whether their orientation is parallel to or perpendicular to the line between the observer's eyes. This may occur because the orientation of the finding chart changes, depending on whether the observations are made in the evening sky or morning sky. The effect (only 0.01 to 0.02 magnitude) is much smaller than the error of the observations, but can be detected by time-series analysis because it is strictly periodic. Its occurrence may depend on the position of the star in the sky, and the orientation of the chart used. It may occur in many stars, but its small size may be swamped by the actual variability of the star, or by the random errors of observation, or by the different circumstances of different observers.

It is obviously not possible to study the behavior of the star on time scales longer than the time span of the data. In fact: as Δt approaches the total time span of the data, there are fewer and fewer pairs of observations with this value of Δt, and the method breaks down because of the statistical need to have several Δ magnitudes in each bin (CP Tau: Figure 5).

If there are large gaps in the data, there may be ranges of Δt with no pairs of observations. For WW Cas, there were no values of Δ magnitude for Δt between 4,000 and 5,500 days.

It is possible that some stars in our sample are multiperiodic, having two or more radial or non-radial periods. Self-correlation is not very effective if stars have two or more periods which are relatively close—i.e., of the same order of magnitude. We plan to use Fourier analysis to study a few of these stars.

Even for stars which show no minima, it is possible to define a characteristic time scale, on the basis of how fast the self-correlation diagram rises to its plateau, i.e., by comparing the rise to plateau with the rise to first maximum in a star that is periodic. To a first approximation, the time scale would be twice the value of Δt at which the diagram reached the plateau.

We have found a continuous spectrum of behavior in the twenty-one stars that we have studied, from irregular, to marginally coherent, to quite coherent. This indicates that the classification scheme for pulsating red giants is arbitrary; there is no distinct boundary between L and SR types. We found a similar situation for low-mass pulsating yellow supergiants: there is a smooth spectrum from periodic W Virginis stars, through RV Tauri stars, to semiregular (SRd) variables (e.g., Percy and Mohammed 2004). In that case, the temperature of the star might be the astrophysical parameter that varies along the spectrum.

In the case of the pulsating red giants, the situation might be more

complicated. Our sample stars vary in temperature, but also in composition type—oxygen-rich or carbon-rich—so there may be at least two controlling parameters. A more detailed study of a larger sample of stars, using both Fourier and self-correlation analysis, would be astrophysically interesting.

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Authors SE, AL, CM, and SW were participants in the University of Toronto Mentorship Program (UTMP), which enables outstanding senior high school students to undertake research at the university. This program has been described by Percy *et al.* (2008). The present paper was presented as a poster at the 2009 Spring Meeting of the AAVSO and the 2009 conference of the Canadian Astronomical Society, as well as at the 2009 UTMP Research Fair.

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Table 1. Self-correlation Analysis of L-type Variables.

Star	*Spectrum*	*Range*	*Period*	*Minima*	Δ*m*	*N*	Δ*t*
U Ant	C5,3(NB)	4.5–7.0	350, 2000	2	0.25	555	23307
V Aps	MB	9.0–11.0	irregular	—	—	142	27414
VW Aql	M5III:D	9.5–12.0	800	6	0.15	2986	25399
UX Cam	M6	7.6–9.5	1000	6+	0.05	1012	13789
AA Cas	M6III: D	7.5–9.5	75	6	0.02	4682	14677
PY Cas	M5III: D	9.5–12.0	irregular	—	—	517	14584
WW Cas	C5,5(N1)	9.5–11.5	irregular	—	—	299	12297
ST Cep	M3Iab:C	7.5–9.0	300-400	4	0.03	1306	26999
AT Dra	M4IIID	5.0–6.6	333, 4000	12, 2	0.03, 0.02	3379	14349
UW Dra	K5pvC	7.0–8.5	360 (?)	3	0.02	3653	33216
GN Her	M4IIID	8.0–11.0	irregular	—	—	237	15172
OP Her	M5II-IIIC	5.5–7.5	75, 650	8, 2	0.04, 0.04	4127	19742
TT Leo	M7D	10.0–12.0	irregular	—	—	419	14663
HK Lyr	C6,4(N4)	7.0–9.5	250	5	0.20	1260	19684
T Lyr	C6,5(R6)	7.2–10.0	400	4	0.04	2667	25348
TU Lyr	M6	9.2–11.5	150	7	0.06	6798	26079
X Lyr	M3.5III:D	7.5–10.5	200, 6500	2	0.03	1573	35004
TY Oph	C5,5(N)	9.0–11.0	irregular	—	—	575	19293
EX Ori	M7III	9.0–11.5	100, 500	4, 10	0.05, 0.60	247	20802
ST Psc	M5D	9.0–11.0	700	3	0.20	843	15013
τ^4 Ser	M5II-III	5.9–7.5	100, 1200	5, 3	0.02, 0.04	5970	19813
CP Tau	C5,4(N)	9.0–11.0	1250	9	0.10	1749	14993
X Tra	C5,5(NB)	5.2–7.2	500	3	0.06	2107	23308

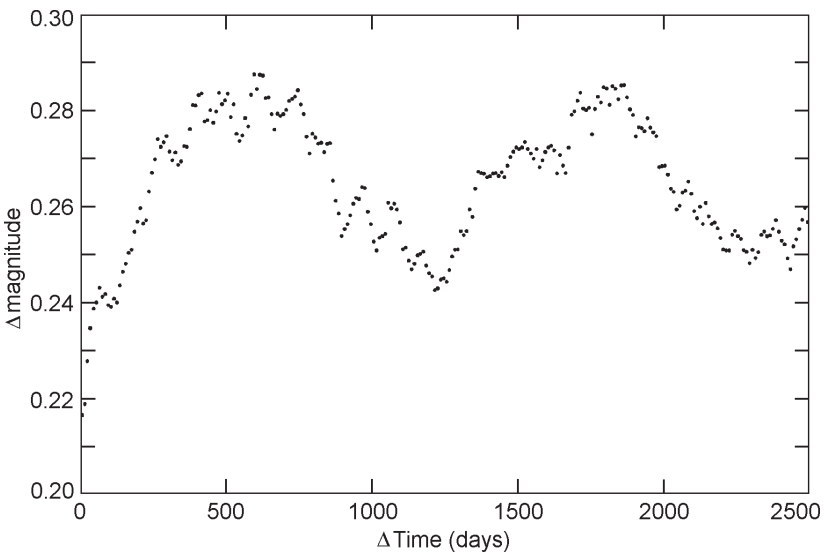

Figure 1. Self-correlation diagram for τ^4 Ser. There are minima at multiples of 100 days, and also of 1,200 days. The former is the primary pulsation period. The latter is an example of a "long secondary period"; its nature and cause are unknown.

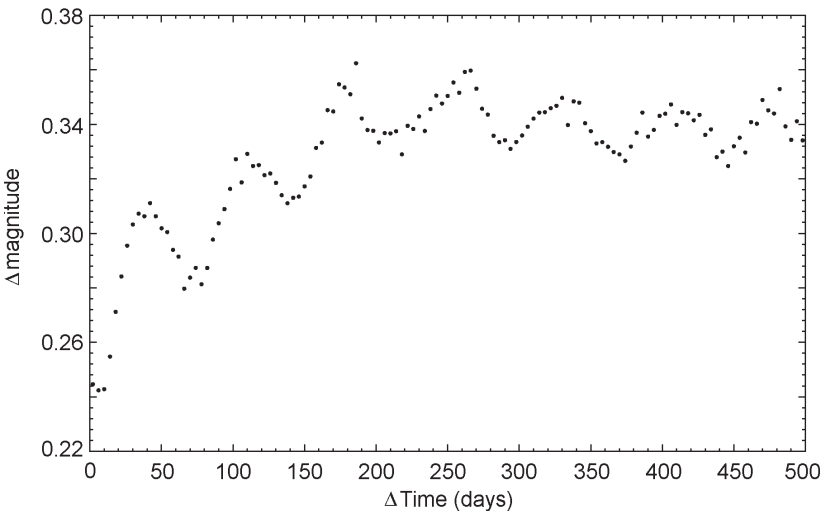

Figure 2. Self-correlation diagram for OP Her. Note the minima at multiples of 75 days. This is a good example of an L star with appreciable periodicity.

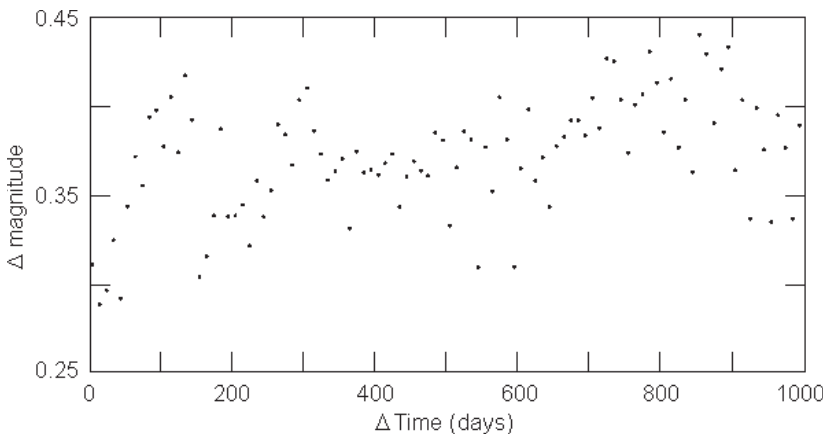

Figure 3. Self-correlation diagram for TY Oph. There are no repeating minima, though the complex structure between $\Delta t = 0$ and 400 days could be caused by one or more low-amplitude periods.

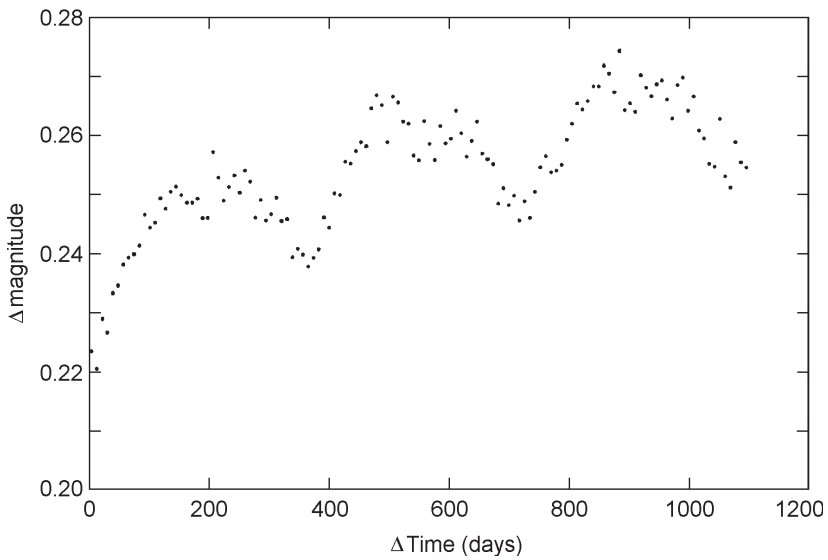

Figure 4. Self-correlation diagram for UW Dra. There are minima at multiples of 365 days, with an amplitude of 0.02 magnitude. These are almost certainly an artifact caused by the visual observing method; see text.

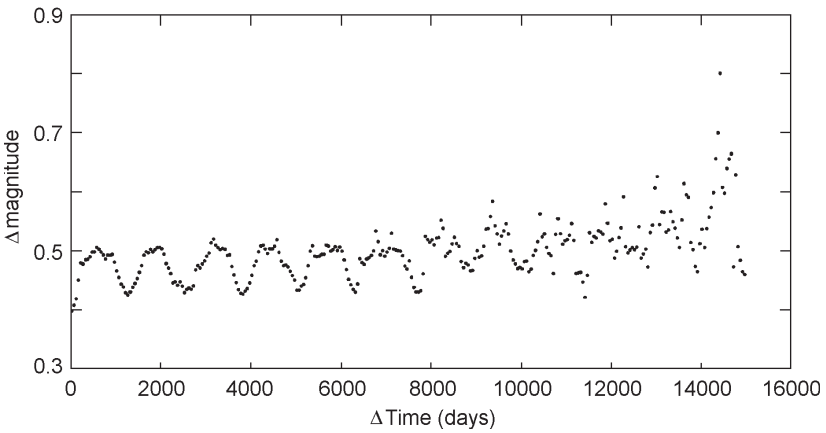

Figure 5. Self-correlation diagram for CP Tau. Note the minima at multiples of 1,250 days—the period. As Δt approaches 15,000 days, the scatter increases, because there are fewer and fewer pairs of measurements, with such large Δt, in each bin.